Compact and precompact sets in asymmetric locally convex spaces

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Let $X$ be a real vector space. An asymmetric seminorm is a sublinear functional $p : X \to \mathbb{R}_+$. If further, $p(x) = p(-x) = 0$ implies $x = 0$, then $p$ is called an asymmetric norm. The main difference with respect to a seminorm is that $p(x)$ can be different from $p(-x)$ for some $x \in X$. An asymmetric metric on a set $Z$ is called a quasi-metric.

If $P$ is a family of seminorms on $X$, then one defines, in the usual way, an asymmetric locally convex topology (LCS) $\tau(P)$ on $X$. This topology is derived from a quasi-uniformity on $X$, so an asymmetric LCS is also a quasi-uniform space (see [6]). The basic properties of asymmetric LCS were studied in [2].

Asymmetric normed spaces were studied in several papers (see, for instance, [5] and the references quoted therein). Some applications of this theory to the study of complexity spaces were given in [7] and in other papers. It is worth to mention that the relations between completeness, compactness, precompactness and total boundedness are much more complicated in quasi-metric and in quasi-uniform spaces than in the symmetric case (see [6] and [3]). A detailed study of compactness and precomapactness in asymmetric normed spaces was done in the papers [4] and [1]. The aim of this paper is to extend these results to asymmetric LCS. Compact linear operators on asymmetric normed spaces, including a Schauder-type theorem on the compactness of the conjugate mapping, were considered in [3]. We shall try also to extend these results to asymmetric LCS.