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BOOK REVIEWS

J.L. Aurentz, T. Mach, L. Robol, R. Vandebril, D.S. Watkins, *Core-Chasing Algorithms for the Eigenvalue Problem*, SIAM, Philadelphia, 2018, IX + 149 pp., ISBN 978-1-611975-33-8.

This monograph appears in the SIAM series *Fundamentals of Algorithms* and has the following rough contents: Ch. 1 Core Transformation, Ch. 2 Francis's Algorithm, Ch. 3 Francis' Algorithm as a Core-Chasing Algorithm, Ch. 4 Some Special Structures, Ch. 5 Generalized and Matrix Polynomial Eigenvalue Problems and Ch. 6 Beyond Upper Hessenberg Form. The work also contains a Preface, a Bibliography containing 75 entries and an Index.

The monograph essentially reformulates and studies the iterative Francis's algorithm as an efficient tool to find the eigenvalues, eigenvectors and invariant subspaces of complex matrices. Each iteration in this algorithm is a unitary similarity transformation which replaces the given Hessenberg form of a matrix by a new one. This one is closer to the upper triangular form which provides the eigenvalues on the main diagonal. The authors introduce an extraneous core transformation and chase this through the matrix until it disappears via a fusion operation. They show why this algorithm works, study its backward stability and apply it for special structured matrices.

The Fortran package EISCOR, which contains eigensolvers based on unitary core transformations, is also provided by authors.

This book offers a well-organized viewpoint on some basic features of solving eigenproblems attached to various complex matrices. With many useful examples treated in detail, it provides a fine independent study text and is suitable for graduate students as well as for researchers interested in solving matrix eigenproblems.

C.I. Gheorghiu

W. Gautschi, *Orthogonal Polynomials in MATLAB. Exercises and Solutions*, SIAM, Philadelphia, 2016, X + 337 pp., ISBN 978-1-611974-29-4.

This book contains a plethora of exercises with detailed solutions, spread on over three hundred pages.

In the first chapter, entitled "A Guide to the Software Packages OPQ and SOPQ", the author provides some very interesting and useful demos concerning the principal functions contained in the software package OPQ (Orthogonal Polynomials and Quadratures). In the subsequent chapters he formulates more than one hundred exercises, presented with their corresponding solutions. The second chapter, "Answers to Exercises on Orthogonal Polynomials", is the largest one; it contains 43 exercises. The third, the fourth and the fifth chapters contains answers to exercises on Sobolev orthogonal polynomials, to quadrature and resp. to approximation. Actually these exercises are very difficult problems, mainly encountered in the author's papers. The solutions are worked out using double-precision accuracy MATLAB routines as well as with variable-precision symbolic MATLAB programs. The author heavily comments on these solutions and distinguishes between various releases of MATLAB with respect to the accuracy of some built in functions.

The work also contains two appendices, A. The Software Package OPQ, and B. The Software Package OPQ (Symbolic Orthogonal Polynomials and Quadrature). It also contains a Preface, the Bibliography, and two indexes (referring to software, resp. subject).

In our opinion the work is important from a twofold reason, namely for the refined mathematics (Numerical Analysis) involved, and also for the expert MATLAB programming.

Orthogonal Polynomials in MATLAB thus serves as both an outstanding text for graduate as well as PhD students and as a rich source of current results for research scientists. Professor Gautschi is a renowned expert in this field.

C.I. Gheorghiu

W. Gautschi, *A Software Repository for Orthogonal Polynomials*, SIAM, Philadelphia, 2018, VIII + 60 pp., eISBN 978-1-611975-22-2. (available only in electronic form)

This impressive document, available only in electronic form, contains the essential information about classical, quasi-classical and non-classical orthogonal polynomials. The classical and quasi-classical orthogonal polynomials share the property that their recurrence coefficients in the three-term recurrence relation are explicitly known. The main aim of this document is clearly stated in Introduction, namely: "Nonclassical orthogonal polynomials, often occurring in specific applications (physics, chemistry, statistics, etc.), are such that the recurrence coefficients are not known in closed form and therefore require special procedures to compute them. This document is concerned foremost with such non-classical orthogonal polynomials."

For classical and quasi-classical polynomials both recurrence coefficients in the three-term recurrence relation are provided in closed form. For non-classical polynomials the first 100 recurrence coefficients to 32 digits are provided separately in two colored graphs. It is also important to observe that accessing an URL suggested by author, the reader can find the nodes (in ascending order) of the Gaussian quadrature formula associated with the weight function and in particular the corresponding Gaussian weights.

Along with the "Orthogonal Polynomials in MATLAB", SIAM (2016), the software delivered by the famous professor Gautschi serves as both a valuable source of current results for research scientists, as well as an outstanding text for various graduate students.

C.I. Gheorghiu

V. Dolean, P. Jolivet, F. Nataf, *An Introduction to Domain Decomposition Methods: Algorithms, Theory, and Parallel Implementation*, SIAM, Philadelphia, 2015, X + 238 pp., ISBN 978-1-611974-05-8.

This well written book is an excellent introduction into domain decomposition methods in a complete and accessible manner. It is structured on 8 chapters, shortly described as follows. 1. *Schwarz methods.* Three different versions of Schwarz algorithms are introduced, namely Jacobi-Schwarz method, additive Schwarz method and restricted Schwarz method. The equivalence with a block Jacobi method is proved. 2. *Optimized Schwarz methods.* Here is introduced the P.L. Lions' algorithm, which is based on Robin interface conditions. The convergence factor is computed and a general convergence proof is given. The Després algorithm for Helmholtz equation is analyzed together with its implementation. Optimal interface conditions, more general than Robin conditions are considered. 3. *Krylov methods.* The advantage of using domain decomposition methods as preconditioners for Krylov methods is explained. Fundamental notions on Krylov spaces are presented in a clear manner. In addition, a comprehensive description of conjugate gradient and GMRES algorithms is provided. 4. *Coarse spaces.* Two-level methods are introduced. The Nicolaides' coarse space (the simplest coarse space) and its implementation in FreeFem++ are provided. 5. *Theory of two-level additive Schwarz methods.* A class of spectral coarse spaces, which represents a generalization of Nicolaides' coarse space is introduced. A theory of these two-level methods is presented. 6. *Neumann-Neumann and FETI algorithms.* A very efficient short description of multifrontal solver for two-subdomain case (for a matrix arising from finite element discretization) is provided. The iterative Neumann-Neumann and FETI algorithms are introduced in a clear and accessible manner using a two-subdomain decomposition for Poisson problem. Their algebraic counterpart and implementation in FreeFem++ are also provided. 7. *Robust coarse space via generalized eigenproblems, The GenEO method.* An adaptive abstract coarse space together with most classical two-level methods are presented. The SOARS-GenEO-2 algorithm is introduced. 8. *Parallel implementation of Schwarz methods.* The use of domain decomposition methods on parallel computers through their implementation in FreeFem++ is discussed. A comprehensive analysis of some numerical simulations of very large scale problems on high-performance computers is presented. The book is completed by a selective bibliography and an index.

Besides the material about domain decomposition methods, we notice that useful notions related to Krylov spaces, Helmholtz equation as well as free finite element package FreeFem++ are also introduced in a didactic and attractive manner.

The presentation of the material is very clear and completed with examples, programs and graphs. We therefore consider that this book is highly recommended not only to advanced computational scientists and mathematicians, but also for anyone involved in numerical solving of the Partial Differential Equations.

Flavius Pătrulescu